

which are controlled by a single, spring-return (center-off) SP3T toggle switch mounted in a hand-held unit on a cable.

I would like to acknowledge the substantial contributions of R. N. Euwema and J. A. Elmgren in developing this experiment.

¹ See, for instance, J. W. M. DuMond and E. R. Cohen, *Rev. Mod. Phys.* **25**, 691 (1953).

² A. C. Melissinos, *Experiments in Modern Physics* (Academic, New York, 1966), pp. 2-8; *The Taylor Manual*, edited by T. B. Brown (Addison-Wesley, Reading, MA, 1959), pp. 392-94.

³ H. Kruglak, *Am. J. Phys.* **40**, 768 (1972).

⁴ The students are also given values of the individual parameters for reference. The parameters $d \approx 9$ mm,

$l \approx 3$ mm, and $\rho \approx 0.9$ g/cm³ were measured by routine methods. The viscosity of air is taken to be $\eta = 1.69 \times 10^{-5}$ kg/m-sec, the handbook value for 23°C decreased by the mean-free-path (mfp) correction (8%) for the 2 μ m-diam drops typical of our observations. Strictly, the mfp correction should be proportional to $\Delta l^{1/2}$; our single value of effective viscosity is in error by about 2% at the extreme fall times commonly observed ($\sim 15, 35$ sec).

⁵ The posting of the original ΔV and Δt values in addition to the computed quantity $\Delta V^{-1} \Delta t^{-3/2}$ gives the instructor a chance to spot gross arithmetic blunders, which occur all too frequently.

⁶ E. Whittaker and G. Robinson, *The Calculus of Observations* (Van Nostrand, Princeton, NJ, 1944), 4th ed., Sec. 101, 104.

⁷ Similar to Experiment EF-1, A. M. Portis and H. D. Young, *Berkeley Physics Laboratory* (McGraw-Hill, New York, 1971), 2nd ed.

Direct "Literal" Demonstration of the Effect of a Displacement Current

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Maxwell's displacement current density is one of the physical concepts—like magnetic induction in Faraday's Law—that requires the freshmen physics students to understand changing flux and circuital line integral paths. These concepts seem to be anomalously difficult to understand. Although the existence of electromagnetic radiation might seem to be an adequate test of Maxwell's theory and a demonstration of displacement currents, one often feels a more literal demonstration of displacement current is needed. This note describes a lecture-demonstration apparatus which shows the effect of the displacement current between two capacitor plates and introduces the student to the motivating idea that an induced circuital magnetic field is produced not only by a real current in a wire leading to a capacitor plate, but also—in the sense of continuity—by the displacement current between capacitor plates.

Our apparatus is simple: A toroidal coil is either placed around a wire leading to a large pair of capacitor plates to demonstrate Ampere's law, or the toroidal coil is inserted between the capacitor plates as shown in Fig. 1 to demonstrate

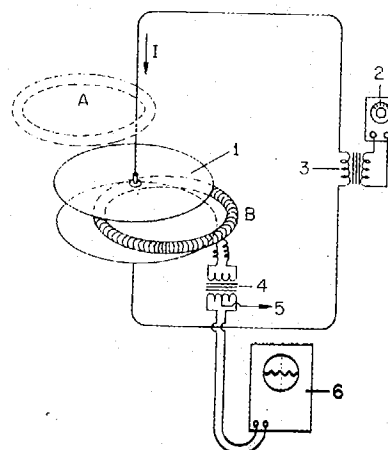


FIG. 1. Schematic of general setup of the demonstration showing toroidal coil in position A for Ampere's law and in position B for displacement currents: (1) capacitor plates; (2) audio oscillator; (3) matching or driving transformer described in text; (4) output transformer described in text; (5) center tap of output transformer grounded to coil shield (not shown) and to cable shielding leading to oscilloscope; (6) differential input, fairly high gain, low frequency oscilloscope.

the effect of the displacement current. A magnetic field produced by the alternating currents of either form in the right hand terms of

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I + \epsilon_0 \iint \frac{d\mathbf{E}}{dt} \cdot d\mathbf{S} \right)$$

induces, by Faraday's law,

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \iint \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S},$$

an alternating voltage which is displayed on an oscilloscope and also on a large lecture slave oscilloscope. The concept is completely obvious to most physicists, but the reader who substitutes actual numbers into the above equations for a suitably sized capacitor and coil and for convenient choices of ac frequencies will find that the induced voltage is inconveniently small. Moreover, if one must contend with a small voltage signal, then the stray capacitive pickup voltage will mask the desired effect. For example, if one uses a 10 000-turn toroidal coil of about a 30–40 cm diam, with a cross-sectional area of about 10 cm², and uses large capacitor plates with a spacing of 6–10 cm that are driven at 60 Hz by a small neon-sign transformer at several kilovolts, then the induced voltage is little more than a few microvolts and the unwanted pickup signal will be more than a few volts. Alternately, if one chooses a frequency which is high, such as 1 MHz, then special equipment such as a radio-frequency amplifier and a fast scope will be required. We believe that the demonstration set-up described in this note comes close to optimal simplicity and, except for the construction of the coil itself, requires no special apparatus that is not commonly available in a physics laboratory or lecture-demonstration stockroom.

Our apparatus is based on a large capacitor with two 80-cm-diam aluminum plates, held apart with a 10-cm spacing. The lower plate is grounded, and the upper plate has a banana plug in the top so that the connection to it may easily be opened and closed. The capacitor is driven from a small powdered iron torus transformer (Ferroxcube 400 T 750) which is wound with a ratio of 50 primary to 1000 secondary turns. This matches the capacitor to a 600 Ω audio oscillator

(Hewlett Packard, Model 200CD or equivalent), so that there is a voltage of one hundred volts across the capacitor at about 20 kHz.

The magnetic induction pickup coil is in the form of a torus and is made as follows: A 238-cm-long, 1.27-cm-radius rubber vacuum hose is wound with 14 000 turns of No. 38-gauge Formvar insulated wire. This can be done by threading a dowel through the tubing and using a hand drill as a rotating device. The wire is glued onto the tube with silicone RTV cement just to facilitate handling. The wire is insulated and protected against abrasion by a larger thin rubber hose (1.27-cm inside radius, 4-mm thick wall) which is slit lengthwise, slipped over the coil and taped securely with mylar tape.

Copper shielding foil is added as shown in Fig. 2(b); the tubing is bent into a circle, excess end rubber is cut off, and the tubing plus shielding is attached to a piece of plywood backing in order to hold it rigidly. The plywood piece has a large hole cut in the middle and has a handle as shown in Fig. 2(a). At the junction of the handle and the torus, the two ends of the toroidal coil are brought to a 1:1 audio-output transformer with secondary center tapped.

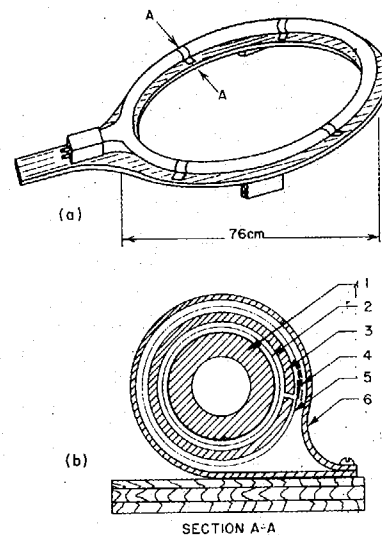


FIG. 2. (a) Toroidal coil assembled on plywood or phenolic board brace. (b) Cross section A-A of toroidal coil showing: (1) rubber tubing form; (2) wire windings; (3) outer insulating tubing; (4) Mylar tape to insulate overlapping portion of (5) copper shielding foil about 0.0025 cm thick; (6) plastic hose or cable clamp.

Probably any audio interstage transistor transformer will do here, but ours was made from a powdered iron Ferroxcube 528T 500 toroidal form wound with 400 turn primary and 400 turn center-tapped secondary. In order to be conservative about stray pickup, the secondary was wound with 200 turns right-handed, and 200 turns overwound left-handed, and the starting points of these two coils were used as the common secondary center tap which was connected to the shielding foil. The secondary of this transformer is led by two shielded RG/59U or equivalent cables to the differential input amplifier terminals of a laboratory type oscilloscope.

It is important to notice that this type of connection, from the coil to the oscilloscope, is essential to the elimination of unwanted stray pickup voltages. The desired signal is push-pull or differential in nature. The small 1:1 transformer helps to reject the common-mode pickup voltage. Common-mode rejection is improved by using an oscilloscope (in our case a Tektronics 503) with a good differential amplifier input with sensitivity of about 1 mV/cm. The shielding of the coil is important, and it also helps greatly to include the small transformer mentioned within the copper foil shielding. The center tap of the transformer secondary is grounded to the shielding.

The demonstration proceeds as follows: The toroidal coil is brought near the wire leading to the upper plate of the capacitor. If shielding is done well there is virtually no signal visible to the students on the oscilloscope. Then the upper lead wire is unplugged from the banana plug connector and reattached so that the toroidal coil encircles it as shown in position *A*, Fig. 1. There immediately appears a several millivolt signal at 20 kHz on the oscilloscope, and as one would expect from Ampere's law, this signal remains completely constant no matter where the coil is held, as long as the wire to the capacitor plate remains inside. To demonstrate that a displacement current also gives an effect, one unplugs the wire to the upper plate, removes the coil, reconnects the wire to the capacitor plate, and slowly inserts the toroidal coil into the capacitor between the plates as shown as position *B* in Fig. 1. The signal now grows continuously until

it reaches the maximum value determined by that fraction of lines **E** or **D** enclosed within the toroidal coil. In our case this was a signal about 60% of the size of the original Ampere's law signal with the wire inside. It is worth pointing out that although this demonstration could be carried out with a portion of a torus or a section of straight coil, such a complete torus makes the demonstration of Ampere's law especially convincing because of the fact that the induced signal seen on the oscilloscope is completely position independent and depends only on whether the wire is inside or outside the loop.

There is a pedagogical flaw or "swindle" in most attempts to demonstrate the effects of "real" fields and this demonstration is no exception. When one inserts a probe into the capacitor as we do here, some of the lines of displacement current go to the coil shield and cause real currents to flow, so that it is not unambiguously clear that the effect comes from displacement currents. However, either the lecturer may make quantitative estimates of the effect to show that it does come primarily from real lines of displacement current through the toroidal core, or else the demonstration will be of value in providing the student with a clear display of the logic and the geometry, though only in a qualitative sense.

If one wishes to pursue the question of displacement currents in this geometry to a greater degree of sophistication, we must remind the reader that the magnetic field which we demonstrate in this "quasi-static" regime (certainly valid at 20 kHz) can be described entirely by the Biot-Savart law if one takes into account all of the real currents that flow, including the radial currents in the capacitor plates. However, the Biot-Savart law itself is derivable from the complete forms of Maxwell's equations. This point is discussed in a few textbooks,¹ and the lecturer must deal with this difficult problem as he chooses.

Variations will certainly suggest themselves. Some may wish to have the radius of the capacitor plates sufficiently small in order that the toroid may be moved continuously so as to encircle first the wire and then the capacitor while showing no change in induced signal. We felt it to be more dramatic not to do so. The use of permeable magnetic material in the toroid would certainly

increase the size of the induced signal. We did not want to do so because of pedagogical simplicity, and also because of weight. Furthermore, we estimated that magnetic nonlinearities and eddy current effects would destroy the dramatic fact of a null signal when the capacitor wire did not pass through the toroidal loop. Finally, a new version could easily be made considerably smaller, but

no harm is done by making the demonstration easily visible to hundreds of students.

The authors gratefully acknowledge several useful suggestions from Marius Isaila.

¹E. M. Purcell, *Electricity and Magnetism*, Vol. 2, Berkeley Physics Course (McGraw-Hill, New York, 1965). Section 7.12 and particularly Fig. 7.31 on p. 262 deals specifically with this point.

Simple Laser Test for Uniform Carriage Motion in Interferometers

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A simple procedure to check for uniformity of advance of the movable element of an interferometer is described in this note. Using this procedure, a record of the uniformity of motion over the entire travel is obtained from which the positions of possible trouble spots may be easily determined.

It is most important that the motion of the carriage on which the movable element of an interferometer is mounted follow the micrometer advance continuously. Often, the carriageways are not completely smooth and/or the carriage gliders become worn. These conditions, separately or in combination, result in an imperceptibly jerky advance of the movable element, with the carriage "hanging up" frequently as it travels. This condition may exist only along a very small portion of the track in an otherwise precise and well-maintained instrument. The results of even a very limited segment being in this condition can be quite serious in terms of the resulting experimental errors. As a practical matter, it is wise to consider an interferometer suspect until proven otherwise.

The following procedure may be used to check the entire travel for uniform carriage motion with micrometer advance. To make the discussion spe-

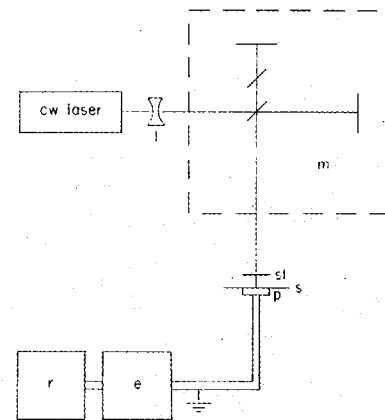


FIG. 1. Schematic diagram of experimental arrangement for test of uniform motion of the movable element in a Michelson interferometer.

cific, the procedure will be explained for a Michelson interferometer. The basic arrangement is shown in Fig. 1; similar arrangements can be used for several other types of interferometers. A low power, cw laser is directed through a diverging lens l and into the Michelson interferometer m . The fringes are projected onto an inexpensive photodiode p mounted behind a screen s and variable slit sl . The screen, which is used for visual alignment of the fringes, permits exposure of the detector through a fixed slit with a width approximately equal to the maximum aperture of the variable slit. The detector system consists of a Keithley 610C electrometer e and a strip-chart recorder r . The recorder drive is attached through a gear box (not shown) to the carriage advance of the interferometer. In this fashion, a plot of the variation in intensity as the fringes sweep